THE MANUAL OF MATHEMATICAL MAGIC

Peter McOwan with Matt Parker

This is your card

Acknowledgments

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The Manual of Mathematical Magic

Mathematics and magic may seem a strange combination, but many of the most powerful magical effects performed today have a mathematical basis. Famous magicians such as Derren Brown and David Blaine use mathematics-based tricks in their shows, but mathematics is also the secret behind the technologies we use, the products we buy and the jobs we will have. Mathematics is the language we use to describe the world around us - it's the basis of all the sciences.

This book will show you how to perform some magical miracles to impress and entertain your friends. But it will also explain the mathematics behind the tricks and how that same mathematics is used in the real world. It also looks at the varied and exciting sorts of jobs that make use of the mathematics powering your magic.

All the tricks in this book are self-working, which means you don't need to know any clever sleight of hand, like dealing cards from the bottom of a deck. But be warned: knowing the mathematical secret isn't the same as being able to perform the tricks well. To do that, you need to use your performance skills to create a sense of wonder. A good presentation is where your audience is pulled into the magic happening and astonishment happens. With an imaginative story, you can take a simple mathematical trick and turn it into a jaw-dropper.

This book will give you some ideas for presentations, but be creative, come up with your own way. Thinking creatively about new ways to solve problems is the key to good magic but it's also one of the key skills of a good mathematician and one of the useful employment skills you get from being good at mathematics. As we will see, many famous mathematicians were also magicians.It's not a coincidence - they enjoyed their maths, and also enjoyed using it to entertain. And now you can too.

As you start to work your way though this book, you will find magic that uses a whole range of mathematical ideas that you may already have come across, from simple addition and subtraction, to prime numbers, geometry, algebra and statistics. We hope that this book shows that all of maths can be exciting, magical and useful. There is even an advanced section where some maths that's probably new to you is introduced, with stunning magic results. So have some serious fun, learn, practice, create and above all entertain.

Some of these tricks are in the acts of professional magicians. So when you do perform them, please remember the Magicians Code: practise, practise, practise. And never reveal the workings of magic tricks to your audience!

The Symbols

Throughout this book you will see four different symbols. Each one is used to explain a different aspect of a trick.

The *magic symbol* is how the trick will actually look to your audience. This will explain how the trick should flow and will give you an overview of how it should come across to observers. This won't tell you the sneaky things that you need to be doing!

This is where the sneakiness is explained. The *presentation symbol* is where the actual trick itself is explained. This will go step-by-step through what you need to do to make sure the trick works and how you can increase the effect on your audience. Don't forget that the best magic tricks use a very simple device but then this is built up through a brilliant presentation. Once you grasp how the trick works, be sure to try expanding on your own presentation ideas!

All of the tricks in the book are based on mathematical principles. Normally this is explained in the presentation section, but when the maths is particularly important, or you need to learn some extra maths to make the trick work, you will see this, the *maths symbol*.

Some of the best tricks that are based on maths are all around us every day, from our computers to the food we eat. Whenever the maths that is used in a trick has some particularly interesting applications to the real world or to different professions, you will see the *applications symbol* and a brief explanation. You can always do some more research into any of these applications and you will definitely find many more than we had room to mention.

So on with the magic. Are you watching closely?

THE MANUAL OF MATHEMATICAL MAGIC

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“Inventing a magic trick and inventing a theorem are very similar activities.”

Mathematician and magician Professor Persi Diaconi

The Magic of Basic Mathematics

Let's start with the easy stuff. Maths is about manipulating numbers and looking for patterns in those numbers. Adding, subtracting, multiplying and dividing are the building blocks of basic maths. We need these skills every day, just like reading and writing. This first section shows that even these simple mathematical steps can be magical with the proper presentation.

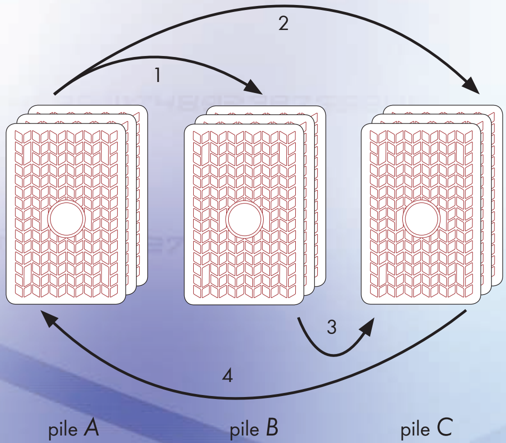
Addition
It All Adds Up To A False Cut

For many of these tricks, you will need to have certain cards in certain places in the deck for them to work. Magicians call this positioning of cards in a deck *stacking the deck*. But, as your audience might suspect you've been up to something sneaky before the trick starts, that's where *false shuffles* and *cuts* come into play. It looks like you are fairly mixing the deck while in fact you do no such thing and the cards are in the same order at the end as they were at the beginning. Here's one of the simplest false cuts. It looks like a real gamblers cut but doesn't change the order of cards in the deck at all.

Put your pack on the table. Let's call this pile *A*. Now cut off about the top third from this pile and place these cards to the right. Let's call this pile *B*. Now cut half of what's left in pile *A* and place this further to the right of pile *B*. We'll call this pile C.

All that remains is for you to pause, then reassemble the pack. Place pile *B* onto pile C, then take this combined pile and place it on the cards in pile *A*.

To the audience this looks like a fair series of cuts, but if you try it with a pack you will discover that the pack is in exactly the same order at the end as it was in the beginning.



Why? Well, all this false cut does is show that when you first cut the cards into piles, you put the top third in pile *B*, the middle third in pile C and leave the bottom third in pile *A*. So, going from left to right, your pack is cut into three piles *A (bottom)*, *B (top)* and C *(middle)*. When you gather them back together, you put *B* on top, then C in the middle and then *A* at the bottom – exactly where they came from!

It's obvious that the order of the cards stays the same, it is just simple arithmetic, but done in an offhand, casual way while telling the audience you really are mixing the cards they will believe you. It is enough to confuse the onlookers, and a useful way to start your tricks too.

It looks like a real gambler's cut but doesn't change the order of cards in the deck at all.

Addition and Subtraction
The Dented Card Trick

You demonstrate that your ‘super sensitive fingertips’ can find the dent in a chosen card which has been tapped by a spectator, even when it happens with your back turned.

First your spectator shuffles the deck. No funny business - it can even be their own battered deck. Then you ask them to choose any number between 1 and 10. You turn your back and tell them to count off the top cards, one by one, until they reach their secret chosen number and then tap and remember the card at that number. They must leave it where it is.

With your back still turned they then replace the cards and give the deck to you. You dramatically turn to face them, explaining that you will shuffle the cards then try and read them with your sensitive fingertips. You will then move their card to a different position in the deck.

Meanwhile, you have casually put your hands behind your back. As you are talking, your hands are doing some secret counting. Rapidly count off exactly 19 cards and as you do, reverse their order, then replace them on the top of the deck. Announce that you have been successful! You found their ‘dented’ card and have moved it to the twentieth position in the deck.

Bringing the pack into view ask your spectator what number they thought of. Suppose they tell you it was 7. You start to count cards out loud from the top of the deck beginning, “8, 9, 10 ...” and so on ‘feeling’ each one as you go. When the count reaches 20, pause. What card did they ‘dent’? They name the card. You turn over the twentieth card and it’s theirs!

So how does it work? Your spectator chooses a number between 1 and 10, let's call it X. Their card starts out in position X. Behind your back, you reverse count off the top 19 cards so the card that was at X from the top is now at position $19-X+1$. (To see why we add 1, imagine they chose $X=1$. After the reverse count off, the chosen card is the top card. It doesn't move to position $19-1=18$, it moves to position 19. We have to add an extra 1 as we begin counting at 1 rather than 0. So their card is now at position $20-X$.)

In the example where your spectator chose $X=7$, after your hidden reverse count, the card is now at position $20-7=13$. Your final countdown actually counts off $20-X$ cards, starting at 7 (the spectator's X). Mathematically, this is written $(20-X)+X$ and of course this is 20. So their card will always end up at position 20 no matter what number they originally chose!

Addition and Subtraction
The Force of Nature

While your spectator thinks they are completely in control, you are able to force them to select a card of your choosing. You can then reveal the value of this card in any clever way suitable. For example, you could have texted or emailed it to them earlier. Get them to read the message now.

Seemingly allowing the spectator a free choice but actually ensuring the card you know is selected, is a basic magician's tool and the basis of many a good trick. It's called a *force*. There are loads of ways to do it but here is a simple mathematical way.

For the first force, secretly put the card you want selected on the top of the deck. Ask your spectator to tell you a number between 1 and 10. Explain to them that their choice was free and that you want them to count down one card at a time to their chosen number.

Before they do this and to help things along, you demonstrate what they need to do and count down that number of cards one at a time onto the table, so reversing their order.

You then scoop up the cards from the table, pop them back on top of the deck and pass the complete deck to the spectator.

Get them to do the counting and their freely chosen card is, exactly the one you knew it to be.

$\sqrt{x}$

This works because of simple addition and subtraction. Say they choose 4. Your example deal of four cards onto the table puts your force card (previously on top of the deck) at the bottom of the tabled pile. Putting this tabled pile back on the deck and letting the spectator deal again reverses that, so the 4th card down now becomes the top card on the pile on the table.

The two deals cancel each other exactly. Done casually, the spectator won't remember that you helpfully counted the cards out the first time so don't remind them.

Leaving a period of time between your first demonstration deal and their later actual deal is called *time misdirection*. Speak to them during this stage. Ask questions: why did you choose 4? Has that some special significance? And so on, to help them to 'forget' your casual useful demonstration. You can even throw in a false cut after you did the first deal to remind them they are counting out using a 'shuffled' deck and consequently to make your powers seem even more amazing.

Clearly, to make this work, you need to know the number they selected. So in the presentations, make sure you say something like, "Give me any number..." rather than, "Think of a number...", unless, of course, you can mind read! You could also start to count down from 10 to 1 and ask them to stop you at any number, or ask them to write a number on the board. Get creative. But get that number!

Multiplication
Showing Double Digit Dexterity

Most people can do a speedy multiply by 10. You just add a zero to the end of the number – 23 multiplied by 10 is 230, simple. Now you can prove your superior mental superpowers by speedy multiplication of any two-digit number by 11. You explain to the audience that this is clearly far more difficult. They have the calculators on their mobile phones ready to check, but you do your calculations correctly before they even start to click the keys.

Imagining the 11 times trick

To give us this superpower, we make use of two things. One is maths and the other is the human brain's power of imagination.

To do a lightning calculation multiplying any two-digit number by 11, you need to use some visual imagery and use your imagination. Let's take the number 52 for example. Now imagine a space between the two digits, so in your minds eye you imagine 5 2. Add the two numbers together and imagine putting the sum of them in the gap in the middle, so you see 5 (5+2) 2. And that is it, you have the answer: 11x52 =5 (7) 2= 572.

The double trouble trick

But what if the numbers in the gap add up to a double digit? For example, suppose you want to multiply 98 by 11. So, you imagine 9 (9+8) 8. But that bit in gap in the middle gives you 9+8=17, so where do you put these digits? 9 (17) 8?

Easy, just leave the second number (here the 7) in the gap as before and imagine moving the 1 up a place, so you have
(9+1) 7 9 = 10 7 9 = 1079. Correct again.

The maths behind this is fairly easy if you explore it. Suppose you have the number *AB* (that's *A* tens and *B* ones) and you want to multiply by 11. First you multiply by 10. That's easy, 10x*AB* = *AB*0 (*A* hundreds, *B* tens and 0 ones). Then you add another *AB* so you've got 11 lots of *AB* altogether, giving you *A* hundreds, (*B*+*A*) tens and (0+*B*) ones.

This is exactly what all that sliding numbers around in your imagination has been doing without knowing it. Of course, if the middle *A*+*B* is more than 10 (ie it's a double digit number), you just slide the first digit up to the hundreds column, and it's sorted.

Lightning calculations by mind power and a maths trick. Mathematicians make use of their imaginations all the time. Our brains are really good at imagining things and creating pictures in our heads. Often that's the way we solve tricky problems or come up with clever visual imagination tricks like this one.

Multiplication and Addition
Doing Fibonacci's Lightning Calculation

On a piece of paper, write the numbers 1 to 10 in a column. You are now all set to amaze with the speed at which you can add ten numbers.

Ask your friend to choose any two two-digit numbers and write the numbers down in the first two spaces of your column, one under the other. Your friend then makes a third number by adding these first two numbers together and writes it below the first two, in effect starting a chain of numbers. They make a fourth number by adding the second and third, a fifth by adding the third and fourth, and so on, until your column of ten numbers is full.

To show how brilliant you are, you can turn away once your friend has understood the idea, say after the seventh number in the list. Now you can't even see the numbers being written.

Meanwhile, with your back turned, you are actually multiplying that seventh number by 11 to get the final answer.

Let's imagine your friend chose 16 and 21 to start with. The list would look like this:

1. **16**
2. **21**
3. **37**
4. **58**
5. **95**
6. **153**
7. **248**
8. **401**
9. **649**
10. **1050**

You now turn round and write the sum of all ten numbers straight away! Lightning quick, you say it is 2728. Let them do it slowly on a calculator to show your brilliant mind skills are 100 per cent correct.

The final answer just involves multiplying the seventh number by 11. Why?

Well this chain of numbers where the next term is made by adding the previous two terms is called a *Fibonacci sequence*. Fibonacci sequences have special mathematical properties that most folk don't know about...

So let's look at the trick. We start with the two numbers *A* and *B*. The next number is *A+B*, the next number is *B* added to *A+B* which is *A+2B* and so on. Going through the number chain we find:

1. *A*
2. *B*
3. *A+B*
4. B+*(A+B)* = *A+2B*
5. (*A+B*)+(*A+2B*) = *2A+3B*
6. (*A+2B*)+(*2A+3B*) = *3A+5B*
7. (*2A+3B*)+(*3A+5B*) = *5A+8B*
8. (*3A+5B*)+(*5A+8B*) = *8A+13B*
9. (*5A+8B*)+(*8A=13B*) = *13A+21B*
10. (*8A+13B*)+(*13A+21B*) = *21A +34B*

Adding up all 10 numbers in the chain gives us a grand total of *55A+88B* – check it yourself. But look at the seventh number in your column... this line is *5A+8B*. It is exactly the total of the chain but divided by 11!

So working backwards, you can get the final total by multiplying the seventh term by 11. And the maths proves this lightning calculation will work for any two starting values *A* and *B*.

It is up to you to present this trick in such a way that it looks like you are just very, very clever. Which of course you are, as you now know how to use a Fibonacci sequence for magic.

Division
The Fast Five Trick

Look all around, fives are everywhere. Five fingers, five toes. Now think about division. Divide by 10? Easy you say – just move the decimal point one place to the left. But by 5? Far more of a challenge! Now you can impress you friends with your ability to divide any number by 5 at super speed - and do the calculation correct to three decimal places!

Not a lot of people know this, but dividing a large number by 5 can actually be done by a very simple two-step method.

Step [1]: multiply the number by 2.
Step [2]: move the decimal point.

That's all there is to it.

Now you can impress you friends with your ability to divide any number by 5 at super speed.

Of course, your claim to do the calculation correct to 'three decimal places' is just presentation to make it sound more impressive. It's really easy. All you do is get the answer, say 23.7, and add two zeros after the 7 to give 23.700. So that's the answer right to three decimal places as you promised. Our maths proof below will actually show us that we will only ever get an answer with one decimal place. The rest is just magical flim-flam!

It is always a good idea to start easy with a simple example to test our method works so let's look at 10 divided by 5. First, our method says we double the number, so doubling 10 we get 20. Then we shift the decimal point, so 20 now becomes 2.0 or 2. The answer's correct. Good start.

Now for a harder example, say 190 divided by 5. This looks tough without a calculator, but following our method, step [1], double 190 to get 380, and step [2], move the decimal point, so 380 becomes 38.0 or just plain old 38.

Harder still: how about 4567 divided by 5. Again, step [1], double the number, so we have 4567 x 2 = 9134. Then step [2], shift the decimal point to get 913.4. It's correct - check with a calculator if you don't believe it.

And lets get silly: 123456789 divided by 5. Step [1], double it, so 2 x 123456789 = 246913578. Step [2], shift the decimal point and the answer is 24691357.8. (You may really want to check this on a calculator!)

Will it always work?

Could there be a number somewhere that you haven't tested and that the trick doesn't work for? That would be embarrassing so let's look at the maths. That way, we can be sure as we can prove it will always work and save red faces all round.

Dividing by five is like doing half of a divide by 10. Dividing by 10 is easy, you just shift the numbers down a 'slot', so 100 divided by 10 is 10.0, 23 divided by 10 is 2.3 and so on.

Well, division has a rather neat trick or *mathematical property* of its own. As you know, fractions are just another way of writing divisions. So 100 divided by 10 can be written as $^{100}/_{10}$ and 23 divided by 10 can be written as $^{23}/_{10}$.

Division
The Fast Five Trick

And if you do the same thing to the top of a fraction as you do to the bottom, the answer to your division (or the *ratio* of the top to the bottom) remains the same.

So multiplying the top and bottom by 2 each time, we have
$^8/_4 = {}^4/_2 = {}^{16}/_8 = 2$. The result is always the same.

Taking any number *A* and dividing by 5 means that $^A/_5$ is exactly the same as $^{(2xA)}/_{10}$. Here we have just multiplied top and bottom by 2.

And there we have it - that simple formula proves the maths. To divide ANY number *A* by 5, first double it, step [1] and then shift the decimal place, that is divide by 10, step [2]. We can see it will always work.

What we have done here is what mathematicians and computer scientists do all the time - no, not divide by 5! They to come up with a series of steps (which we call an algorithm) and then try to show mathematically that it will always do what is supposed to.

A Challenge

Try a bit of maths yourself. Here is a two step algorithm to multiply a number by 5:

Step [1]: divide by 2.
Step [2]: move the decimal point to the right.

Can you work out if this will method will always work?

Factorising
The Calculator Beating Trick

You start by asking for a random four-digit number from anyone in the audience. You write it up on a board. Now, you explain, in a moment the next volunteer will give you a second four digit number and you will multiply them in your head faster than anyone else can, even if they are using a calculator.

But wait! If they already know the next number, they can start the calculation early! To make it fair, as soon as they say the second number, you will also pick a new four-digit number and multiply the first number by it.

To make it even harder, you will perform both multiplications and then add the answers together, to get the total faster than anyone with a calculator. And sure enough, you do!

Make a big deal of the fact that you will be multiplying two random four-digit numbers faster than a calculator. Then play down everything else that you add to the calculation, make it look like an after-thought. Afterwards, people will only remember that you multiplied together random numbers given by the audience.

Begin by writing the first number from the audience on the board. Indicate below where the next number will go, write up a multiplication sign next to it and show where you will write the answer. Then, to the right, write the first number again, with a new multiplication sign and indicate where you will put your random number and then that answer. Where the two answers go should line up so you can put an addition sign between them and an equals sign below.

As soon as the volunteer gives you the second number, write it in its place and then write your number in its place.

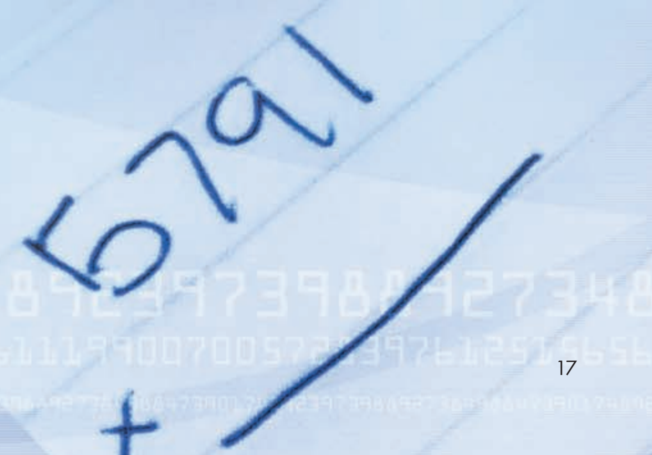
The trick here is to choose your number so that each digit is the *nine complement* to the corresponding digit in their number. This means chose the digits that, when added to the digits in their number, give a total of nine each time.

So if their number starts with a 1, you would start your number with an 8 because 1 + 8 = 9. Then continue this for all the digits. So if they said 1705, you would choose 8294 for your number.

Then you skip the multiplication answers and go straight to writing the grand total at the bottom.

Let’s say that the very first number that your volunteer suggested was 5791. Write this number in the final answer position, but subtract 1 from it, so in this case it would be 5790. Quickly follow this with the four digits that would be its nine complement, so here you would write 4209.

The answer will always be eight digits long, so write the first four with a space like this: 57 90 and then seamlessly complete it as 57 904 209. It looks more impressive - and hides where the number came from - if you write it with the spaces in the correct places. Then all you have to do is wait for the people with calculators to catch up and confirm that you are correct!



Factorising
The Calculator Beating Trick

Let's say that the very first number that you wrote on the board was *A*. Then you are going to get a second number from the audience that we'll call *B* and you get to choose the third number C. The calculation you perform will be multiplying *A* by *B* and then adding it to *A* multiplied by C. We can write this as $A.B + A.C$.

As you can see, *A* is a *common factor*, so we can simplify the calculation by factorising it out like this: $A.(B + C)$. The audience gets to choose *B* but this doesn't matter as you can pick C to make the total $B + C$ to be a really easy number to multiply by.

Now, you could choose C so that $A + B = 10\,000$ and then write the answer as *A* followed by four zeros. But everyone in the audience will immediately spot how you got the answer and they won't be the least bit amazed.

To hide the trick, we choose C so that $B + C = 9999$ (which is why C is the *nine complement* of *B*). Then the grand total of $A \times 9999$ is one less than *A* followed by its *nine complement*. Try a few different numbers to make sure you are happy with how it works.

With a bit of practice, you'll be able to write out *nine complement* faster than people can even type numbers into a calculator! But be careful to only perform it once in a show, otherwise the audience might start to spot the patterns.

This trick actually works with numbers of any length, as long as they all have the same number of digits. But only using three digits looks too easy and for numbers of five or more digits, it becomes increasingly obvious that the first number appears at the start of the answer.

Addition, Subtraction and Psychology
The Teleporting Card

Teleporting is a staple of magic. First something is here, then it's magically there. In your demonstration you count out exactly 10 cards. You place five of them on one side of the room and the remaining five on the other. Then, invisibly, one card travels from the first pile to the second. Have you really mastered powers of teleporting, or is it all in the mind of your spectators? Here's how this cunning bit of mind manipulation is possible.

This trick uses really simple mathematics and a bit of a psychological brain trick. Start with an ordinary pack of playing cards. In stage one you are going to do what magicians call the *convincer*.

You count out ten cards face down, but you do it in reverse. First card down, count *10*. Next card is 9. Count out loud and clear: *8, 7, 6, 5, 4, 3, 2, 1*. When you put the cards down, spread them slightly so the spectators can see it's a fair count of 10 cards. Convince them - it's true, there are 10 cards on the table. Your spectators are now primed to believe that reverse counting is an accurate way to count cards. Pick up these ten cards, and put the rest of the pack aside.

An action repeated that looks like the previous action is treated much the same.

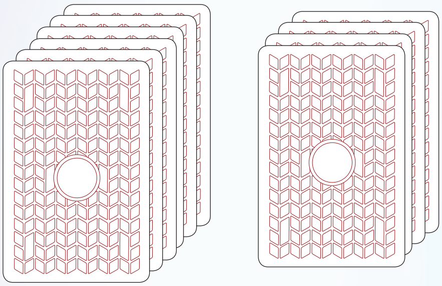
Now comes the sleight of mind. Like a sleight of hand where magicians move cards in a sneaky way to fool their audience, you're going to fool the spectators' brains. You tell them that you're going to deal the ten cards into two piles of five, but again you're preparing them for what's to come.

Into one pile you count from *10* down to *5*, this time not spreading the cards in the pile. That's half the cards, right? You must have five undealt cards left too, because we all know *5+5 =10*. Remind your audience that the two separate piles have exactly five cards each.

Now you make the magical, invisible flight of one card happen. Wave your hands or click your fingers. Perhaps you might even cover the two piles with a cloth and lay your hands on the top of the now hidden piles. Be creative and sell the effect.

Sure enough, when your spectators count the cards in the piles, one pile has six cards, the other has four! Yipes! How did that happen?

If you've read the description so far and still think it's impossible, then you're making exactly the same mistake you're expecting your spectators will. Of course, you're being fooled by the words only, but they will be fooled by your actions too.



10 - 5 *remaining cards*

Addition, Subtraction and Psychology
The Teleporting Card

$\sqrt{x}$

Brain slips and maths errors

Rewind – where is the magic? The convincer says that if you count back from 10 to 1, you have ten cards. That's true in this case.

Now look again at the instructions: the first pile of 'five' cards is created by reverse counting, starting with the ten cards, but counting from 10 to 5. Let's run that again: *10, 9, 8, 7, 6, 5*. Look – that is actually six numbers. The actual number of cards you've counted is six!

So all that's left for the second pile is four cards. Due to the reverse count, two piles of five are actually one pile of six and one pile of four – trick done! The rest is presentation.

What's happening here is called a *cognitive slip*. Human brains love to find patterns, and once they have a pattern that's comfortable, they like to take a break. What happened before will probably happen again, they think. After all, our brains are busy all the time, so they have to use some shortcuts to save on processing power. An action repeated that looks like the previous action is treated much the same.

When you do the convincer, reverse counting from 10 to 1 gives ten cards, showing that reverse counting is correct and establishing a counting pattern. Spreading the cards helps to fix this in people's minds. When the 10-to-5 reverse count of the first pile happens, the idea you established takes over from the arithmetic reality that 10 - 6 = 4.
It's so easy to be tricked. We all know from way back that 5 + 5 = 10, and the reverse count went from 10 to 5, and you say there are two piles of five, so our brains end up fooled.

What's happening here is called a cognitive slip.

Even and Odd Numbers
The Piano Trick

This classic of card magic allows the magician to make a single card change places under very mysterious and possibly musical circumstances.

A volunteer sits at a table and places their hands in front of them as if they were playing the piano. From your pack, you take two cards, and place them both between the spectator's third and little finger on the left hand saying, "Two cards, that's one pair."

Then you place another two cards between the third and second fingers saying, "Two cards, that's another pair." Next you put two cards between the first and second fingers saying, "Again, two cards, that's another pair." Lastly for the left hand, you place two cards between the spectator's forefinger and thumb saying something like, "Two more cards, another pair here as well."

Then on to the right hand. You do the same, taking two cards and slotting them in, repeating, "Two cards, another pair", for each finger slot. Move over the hand, but when you get to the final right hand thumb and forefinger, you take only one card. Pop it in stressing, "Only one card, an odd card here."

What goes in must go out. Pair by pair, starting with the left hand, you take the pairs of cards out, and separate them one into each of two piles, side by side. With each pair you say, "Here is a pair" then separate them onto the two piles.

Continue from the spectator's left hand to the right hand, pair by pair, two cards separated into two separate piles until you come to the all important remaining single card. Hand this odd, single card, to your volunteer and ask them to place it on top of either of the two piles. Summon up all your magic acting skills. You simply tap the pile the volunteer placed the final single card on, and say that this single card will magically and invisibly move across to the other pile.

Pick up the pile to which the single card was added. Count through, separating the cards in that pile into side by side pairs saying each time you count off two, "Here's a pair." Yippee, there are four pairs of cards so the extra card has vanished! To prove this you separate the cards in the other pile in the same manner, pair by pair, but there is a single card left over, so the odd card must have jumped unseen across to the other pile!

$\sqrt{x}$

Well, it could be invisible transporting of a card by magical powers. But, of course, it isn't. This trick works because in reality your volunteer is holding four pairs of cards in their left hand (4 x 2 = 8) and three pairs and one odd card ((3x2) +1 = 7) in their right hand. You have fooled them with a bit of *linguistic legerdemain* – that's fancy talk.

The paired cards, four from the left and three from the right are then divided into two piles, forming two identical piles containing seven cards (an odd number). But because all the way through you've been stressing, "Here's a pair", your volunteer hasn't noticed. They assume both piles are even.

Yes, it's another mind slip to cover the simple maths. With two piles of seven cards, adding the last single card turns that pile into an eight. So when you count them off two by two they are a full set of four even pairs. The spectator wonders, "Where did that odd card go?" The pile where the extra single card wasn't added is still seven so it shows three pairs and one left over. That's the odd card that your spectator assumes (wrongly) must have flown from the other pile.

Once again, this trick shows how easy it is to be fooled if you can't do simple mathematics and believe everything you're told.

Professional magician 'Fitch the Magician', or to give him his proper title, Dr. William Fitch Cheney, Jr. earned the first mathematics PhD awarded by the prestigious American University, the Massachusetts Institute of Technology (MIT) in 1927.

Mathematician and computer scientist Brent Morris was fascinated by magic and card shuffling. He went on to gain a PhD in the mathematics of card shuffling and holds US patents on computer software designed with shuffles.

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12345 \times 8 + 5 = 98765$$
$$123456 \times 8 + 6 = 987654$$
$$1234567 \times 8 + 7 = 9876543$$
$$12345678 \times 8 + 8 = 98765432$$
$$123456789 \times 8 + 9 = 987654321$$

Counting up to countdown

Computer games pioneer and second generation astronaut Richard Garriott performed the world's first full magic show in space while aboard the International Space Station in 2008.

Basic Mathematics
The Applications

These tricks are based on applications of basic mathematical skills: addition, subtraction, multiplication and division. The presentations and the mind games just hide these. Almost every job you could think of needs you to be able to do basic mathematics, whether it's working in a shop, being a plumber, checking your tax return as a super model, working out your fees as a lawyer, running a multinational corporation - or simply calculating your shopping budget as a cash-strapped student.

Binary Numbers
The Super Memory Experiment

You hand a pack of cards (no jokers) to your friend to shuffle them thoroughly to ensure there is no set-up. Get them to split the pack and divide it roughly into two. They then choose which part of the pack to give you. You spread these cards briefly on the table for about five seconds, stating you are 'flash memorising' the order of the pack. Then you collect them up and place these cards out of sight in your pocket.

Your friend then names any card in the pack. Say they name the 10♦. You think carefully and state that card isn't in your pocket. But instead, to prove you've memorised the deck order, you will pull out a card of the same suit as their freely chosen card.

You put your hand in your pocket, murmuring, "Hmmm, 12th card from the top is the 8♦, I seem to remember." And you pull out the 8♦. Just as you said, you've matched their suit with a diamond.

"Of course," you continue, "there was a one in four chance of getting a Diamond at random, so I will pull out another card to add to this card to make the value of your chosen card, the 10♦." Without much effort you remember out loud, "There was a 2♥ five cards from the bottom."

You pull it out to complete your card memory challenge. 8♦ and 2♥, that makes 10, proof you have amazing skill to memorise cards in an instant. Or is it?

Of course it's a trick, and it needs a bit of set up. First, you secretly need to take four cards from the pack. These are the Ace♣, 2♥, 4♠ and 8♦. These are your secret stack and you need to memorise these cards, but that's easy! The order of the suits is simple if you remember the word CHaSeD, the capital letters representing the order of the suits, Clubs, Hearts, Spades and Diamonds.

CHaSeD is an example of a mnemonic, a memory aid, and such word tricks are often used to help in real memory competitions or in studying for exams. For example, you may have come across the phrase *My Very Eager Mother Just Served Us Nine Pizzas* as a way of remembering the order of the planets (but now without Pluto!). As we will see later in this book, mnemonics can create some other amazing tricks.

You take the stack of four cards and secretly hide them in your pocket before your performance. Your friend won't notice the pack is missing these cards when they shuffle and split it. Taking the half of the shuffled pack they give you, just pretend to instantly memorise the cards - don't worry, you really don't need to remember them. Put the half pack in your pocket but make sure that the four hidden stack cards go on top of the pack and that you don't disturb their *CHaSeD* order.

Ask your friend to name any card. They could either mention one of your memorised stack of four cards, in which case you're home and dry with a stunning effect - you just act like you remember where in the pack it is and then pull it out from those top four cards, and bingo! But if they don't mention one of your secret stack, well, that's where the binary number magic comes in...

Binary Numbers
The Super Memory Experiment

$\sqrt{x}$

Binary addition
The first part is easy. For any card named you can pull out the card in the secret stack with the same suit, just remember *CHaSeD*.

But what about the value? For the trick you need to be able to create any number value from Ace (1) through to 10, Jack (11), Queen (12) King (13). The four specific stack cards allow you to do that, whatever the number.

Look at the cards in the stack. You have a 1, 2, 4 and 8. These are the decimal values of the first four *binary base* numbers: $1 = 2^0$, $2 = 2^1$, $4 = 2^2$ and $8 = 2^3$. These allow you to represent all binary numbers from *0000* to *1111*, that's equivalent to all decimal values between 0 to 16 and here all you need is values 1 to 13 for any card named.

This binary representation using *1s* and *0s* is how values are stored in a computer. Here you are storing the values of all possible selected cards in your secret stack of cards.

Build it in binary
In this trick, the values of the cards are converted into binary. Say your friend chooses the 6♦. In binary, 6 is written as *0110* ie no *eights*, one *four*, one *two* and no *units*. You'll first of all pull out the 8♦ to show the suit. But to get the value 6, the binary code tells you that you need a four and a two, but no unit. So you'll need to pull out the 2♥ and the 4♠ in addition.

You either include the suit card in the count if you need it or you keep it separate. Your audience doesn't know in advance what you're going to do, so using the suit card or not using it in the total won't matter to them.

More examples
If the card selected is the King♣, bring out the suit first, that's the Ace♣. Convert the value to binary. A king is worth 13, which is one *eight* plus one *four* plus one *unit*, written as *1101*. So pull out the 8♦ and the 4♠ to add to the Ace♣ on the table.

If the card is the 4♦, pull out the 8♦ for the suit, then the 4♠ for the value. In this case, you don't add the suit card to the other cards to make the value.

Practise so you are confident with working out the set of cards you need for each selection, and practise the patter or dialogue as that's what sells the trick. Remember, this is a great feat of mental powers, so make sure it looks like it.

You could present this as a psychic experiment where you claim you can read the printing on the cards with your fingertips when they are in your pocket, and then you don't need to pretend to do the card memorisation. Whatever presentation you decide on, ensure that you explain clearly to the spectator what you're doing, finding the suit then the value, and the impossibility of it. Then you have a cool trick to impress with.

"Binary is as easy as 1, 10, 11."

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
16	10000

"There are 10 kinds of people in the world, those who understand binary and those who don't".

Binary Numbers – The Applications

Binary numbers are what make computers work. Computer scientists who develop new software need to understand how a computer uses its binary *1s* and *0s* to efficiently calculate the steps in a computer program. The maths of binary numbers help us run computer programs to stream movies on the web, route the calls from mobile phones, predict tomorrow's weather or even calculate the extent of climate change.

Ternary Numbers
The Card at any Number Trick

A volunteer from the audience gets to select any card they want from a pack of 27 cards and then shuffle it back in thoroughly. Then you will go on to explain the mathematics behind how it could be possible to work out which card it is - only for their card to jump magically to the position of their favourite number!

Start by counting 27 cards off a normal shuffled deck of cards. Whilst you're doing this, have a chat with your audience. Explain that 27 is your favourite number at the moment and maybe have them guess why it is your favourite. Mention that it's because 27 is a cube number (3 x 3 x 3) and then ask them, in an off-hand manner, what their favourite number between 1 and 27 is. Have a volunteer select a card, show it to the audience, and then shuffle it back into the 27 which they return to you.

It's always a good idea to have a volunteer show their selected card to someone else, if not the whole audience. It's an insurance policy against getting to the end of an amazing trick only for them to have forgotten what their card was!

Then proceed to say that you're going to explain the maths behind this trick as you go. This works really well if you've done a few other tricks already that you have refused to explain. Holding the pack face-down, begin taking a card at time and placing them left to right in three face-up piles, starting at the left each time. As you start doing this, explain that it is a common magic trick to put the cards into multiple piles and then get the volunteer to just say which pile their card went into, but not what the card is.

You are going to do this three times. At the end of the first time, pick up the three piles and put them back into a single pile, still face-up. Explain that you were memorising the cards and you now know that their card is one of nine possible cards instead of 27.

Turn the pack over, and deal the cards face up into three piles for a second time. Explain that because these cards have been divided over three piles, you now know one in three what their card could be. At the end of the final time through, explain that if you have been paying very close attention, you now know exactly what their card is – it's all a simple matter of repeated division!

However, you continue to explain, you couldn't have known what their card was until the very end. Ask them now to say what their card was. Holding the pack face-up, count off as many cards as their favourite number and that will be their card!

So the big question is, how did it get to the position of their favourite number?

Ternary Numbers
The Card at any Number Trick

$\sqrt{x}$

All the talk about memorising the cards and reducing the options for which card it could be is completely diversionary. You're not doing any of that at all! Importantly, what you are doing is paying very close attention to how you recombine the three piles each time to form a new stack of cards.

There are three times that you recombine the individual piles, and each time there are three positions where you could put the pile which they have indicated contains their card: the top, middle or bottom.

Here is a table that you will need to remember:

	1st Recombination	2nd Recombination	3rd Recombination
TOP:	0	0	0
MIDDLE:	1	3	9
BOTTOM:	2	6	18

You will use the numbers in the table to work out how to get the card into your volunteer's favourite number position. Suppose their favourite number is 17. First of all, take away 1 to work out how many cards you will need on top by the end of the trick – here it will be 16.

Now you need to work out how to make 16 using the numbers in the table. You will need the 1, the 6 and the 9 ($1 + 6 + 9 = 16$). The table tells that in the first recombination, you must put the pile with their card in the MIDDLE. On the second recombination, you must put it at the BOTTOM. And in the third, you must put it in the MIDDLE again. Just like magic, their card will be the seventeenth one! Only it's not magic, it's base-3 numbers...

Base-3 numbers

You will have already seen a magic trick in this book that uses *base-2* or *binary numbers* instead of the normal base-10 numbers that we are so familiar with. This trick uses *base-3 numbers*, which are called *ternary numbers*.

Base-10 numbers have a *units* column, a *tens* column, a *hundreds* column and so on, each new column being a multiple of 10 of the previous column. In *ternary*, we still start with a *units* column, but then each new column is a multiple of 3 of the previous column. So it goes: *units* column, *threes* column, *nines* column and so on. Then instead of the ten digits in base-10 (*0, 1, 2, 3, 4, 5, 6, 7, 8,* and 9) we only use three digits in ternary (*0, 1* and 2).

To convert the volunteer's number to ternary, start by working out how many *nines* you need. For 16 you only need one *nine* with *7* left over. From what is left over, work out how many lots of *three* you need. Here, *7* needs two lots of *three* with 1 left over. This final number left over is the number of *units* you require. So 16 in ternary would be written *121*.

We can now think of the three recombinations being the *units*, *threes* and *nines* columns of our ternary number:

	1st Recombination UNITS	2nd Recombination THREES	3rd Recombination NINES
TOP:	0	0	0
MIDDLE:	1	1	1
BOTTOM:	2	2	2

It takes a bit of practice to be able to convert a number into ternary as you are doing the trick, but you can stretch out the delivery to give yourself more time. A simple way to do this is to deal the cards slowly when putting them into the three piles. Once you have mastered ternary numbers, this is an amazing trick that stuns both magicians and mathematicians!

Algebra
The Brain Control Experiment

Here is an experiment that looks at the power of the human mind to control distant events. Try it with your own deck of cards first before you look at the explanation - you will even fool yourself! You need one deck of playing cards and one brain.

Give the cards a good shuffle. Spread them the table face down. Now think of the colour RED and select any 13 cards. Shuffle again if you wish, then think of the colour BLACK and select another 13 cards at random. Take both sets of selected cards and turn them face up in a single pile, keeping the rest of the cards in a face down spread.

Now the remote control starts. Concentrate. You are going to separate the cards you selected (and that are now in your face-up pile) into two piles, a RED pile and a BLACK pile, in the following way.

Go through your face-up cards one at a time. If the card is RED, put it in the RED pile. For each RED card you put in your RED pile think RED and select a random card from the face-down cards on the table without looking at it. Put this random card in a pile face-down in front of your RED pile.

Similarly, if you come to a card which is BLACK, put it face-up on your BLACK pile. Think BLACK and select a random face-down card. Put this face-down card in a pile in front of your BLACK pile. Go through this procedure until you run out of face-up cards.

You now have the following: a RED pile and in front of that a pile containing the same number of face-down cards you selected while thinking RED. You also have a BLACK pile in front of which is a pile of random cards you selected while thinking BLACK.

Interestingly your thoughts have influenced you choice of random cards! Don’t believe me? Look at the pile of random cards you chose and put in front of your RED pile. Count the number of RED cards in this pile. Remember that number.

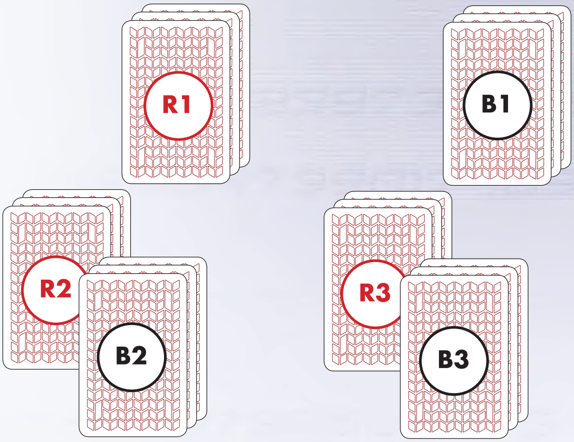
Now look at the random cards in front of your BLACK pile, and count the number of BLACK cards you selected. They are the same! Rather than two random numbers of RED and BLACK, you selected the same number of RED and BLACK cards totally at random because your brain was controlled!

This is a final proof that your sub-conscious mind can make you choose random cards to balance those numbers! So you have done our remote control brain experiment. Is mind control a reality?

Of course it's not mind control. It's mathematics, but you knew that didn't you? But how does this mind-reading miracle work? Well, it's all down to Abracadabra algebra...

Algebra
The Brain Control Experiment

Let’s call the number of cards you dealt R1 for the RED pile and B1 for the BLACK pile. The two other piles in front of these contain a random mixture of red and black, so let’s say that the pile in front of R1 contains R2 reds and B2 blacks, and the pile in front of B1 contains R3 reds and B3 blacks. See the picture.



In the first part of the experiment, what you are actually doing is dividing the pack in half. You dealt two piles of 13 cards each, 26 altogether, half a pack of 52 cards.

Now we also know that in a full pack of cards half are red, and the other half are black, so all your red piles must add up to 26 cards and similarly for the blacks. Writing this mathematically we get:

Equation (1): R1 + R2 + R3 = 26
Equation (2): B1 + B2 + B3 = 26

We also know the number of cards in the RED pile R1 is the same as the number of face down cards placed in front of it (made up of R2 red cards and B2 black cards). So together R2 and B2 must have the same number of cards as R1. Similar reasoning holds for the cards in front of the BLACK pile. So:

Equation (3): R1 = R2 + B2
Equation (4): B1 = R3 + B3

So if we substitute Equation (3) in Equation (1), we can eliminate R1 to get:

Equation (5): (R2 + B2) + R2 + R3 = 26

Similarly if we substitute Equation (4) in Equation (2), we can eliminate B1 to get:

Equation (6): (R3 + B3) + B2 + B3 = 26

As Equations (5) and (6) both add up to 26, we get:

(2xR2) + B2 + R3 = 26 = R3 + (2xB3) + B2

We can subtract R3 and B2 from each side. That leaves:

2xR2 = 2xB3

Finally, we can divide both sides by 2 giving:

R2 = B3

So what the maths shows is that the number of RED cards (R2) in front of the RED pile *must always* be equal to the number of BLACK cards (B3) in front of the BLACK (B1) pile. That is how the magic works. Maths.

In the card trick, we don't know how many cards will be in the piles. Yet by using letters to represent the numbers (these letters are called variables because, well, they can be variable) we can work through the maths, simplifying the expressions to find the answer. The algebra proves the numbers R2 and B3 will always be the same: so long as you follow the instructions for the trick it will always work. The rest of the trick is just as we have done before, presentational flim-flam ... but don't tell anyone how it works!

Algebra
The Number of Matches Prediction

Let's do a trick with coins or matches or even cards if you don't have anything else. In this effect, even with your back turned, you can reveal the exact number of things that a spectator chooses freely from a pile in a stunning three phase prediction.

Build this up. You have a stunning three stage prediction, each stage seemingly more impossible than the last. Take time to build up, chat a bit to misdirect the audience so they forget the exact details of the earlier part of the prediction, and, if you act amazed at each step they will react the same. This trick is known in some magical circles as *The Trick that Fooled Einstein*. If you present it well, it will fool anyone!

Let's assume you have a pile of matches. Ask the spectator to grab a small random number of matches from the pile and hide them, while your back is turned. They don't know how many and so, of course, you can't know either. You then turn round and take a bunch of matches, secretly making sure you take more than the spectator. You told them to take a small number so you take a large number.

You then count your matches and work out what your *target number* should be. Choose a small number to be your *difference number* to take away from your total match count. Say for example you have taken 17 matches. You choose say 3 to take away from this total, to reach your *target number* which in this instance will be 14.

You would turn to your spectator and announce, or write down on a bit of paper, "I've got as many matches as you, then an extra 3 more. And then just enough left to make your number up to 14."

With the prediction made, it's time for the spectator to count their matches. You then count yours out and your prediction is proven true. Say they have seven matches. From your pile, you count out seven matches and put them aside. Part one of your prediction is true: you have as many matches as them.

You then count off your *difference number* of matches off to the side from what's left of your pile (in this example, it would be three). That's Part two of the prediction correct - you have three more matches than they had.

In your pile you now have 17-7-3 =7 matches left. Count your remaining seven matches onto their pile of seven matches, starting counting, "8, 9, 10 ..." up to the total of your stated *target number* which was 14. Part three of your amazing prediction is correct.

The trick here is the three part prediction - it camouflages the simple maths going on. Say you choose X matches, the spectator chooses Y matches and you decide the *difference number* should be Z.

Part One of your prediction is that you have more matches than them, ie $X>Y$. This is true because they took a small number of matches and you took a large number.

Part Two was that you had a particular number Z more matches than them. If you chose your difference number sensibly in the first place, you should have Z to spare, leaving you with $X-Y-Z$ matches.

Part Three is the clever bit. You now add the matches you have left to their Y matches. So you count $Y+X-Y-Z = X-Z$ – exactly the target number you promised!

Algebra wins again, hidden in your prediction. It's really quite simple and will always be true.

Algebra and Addition
The Amazing Coincidence

You claim that some things are simply meant to be, and set about proving it. You need your friend to help you create a 'random' target number. But in fact, you can predict what the target number will be, whatever numbers your friend gives you. The total is... 1089.

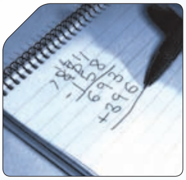
You get your friend to secretly write down any three-digit number. To make it 'more impossible', you say the digits must all be different and the biggest digit must be at the front. You say it's still too easy. Now you want them to jumble things up a bit. You get them to reverse their selected number and write it underneath the first number. They should then subtract this lower number from their first number. Finally, to make it even harder still, have them write their answer backwards and add this new reversed number to their answer. After all of this, you now have a random number even your friend couldn't have predicted in advance. But you can, as you reveal your previous prediction of this amazing coincidence!

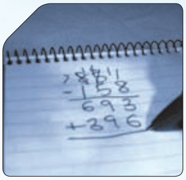
As you will see, the maths shows that the trick will always work if the spectator does what's expected of them and follows the rules. Mathematically we call the things the spectator must do, like choose different digits and put them in the right order, the *constraints*. In this trick it's the presentation (where we say 'lets make it harder' and so on) that makes sure these constraints are followed. If we let things go astray and, for example, let the spectator choose digits that were the same we would have a *constraint violation* and it would upset the maths. Not a good idea!

This trick blends simple arithmetic and a bit of algebra. Let's represent the chosen number as letters *ABC* (that's *A hundreds*, *B tens* and *C units*). So, for example, if the chosen number is 257 that's *A*=2, *B*=5 and *C*=7.

Following the instructions your spectator has to reverse this number, so they will get *CBA*. Next we ask them to subtract *CBA* from *ABC*, which we can write like this:

Hundreds	Tens	Units
A	B	C
- C	B	A





Now you do something sneaky. If you add 100 to ABC and then take away 100, then you're still left with the same number, ABC, right? Well, now imagine the second 100 which you added on was made up from nine tens and ten units (ie 90+10). It makes no difference to the final value of ABC, but it changes our table to

Hundreds	Tens	Units
(A-1)	(B+9)	(C+10)
- C	B	A

Doing the subtraction, we get:

(A-1)-C	(B+9)-B	(C+10)-A

So here is the first bit of magical prediction: the centre number will always, always be a 9! But you can also see why it's important to have all the digits different and with the largest digit first. It's not to make it harder - that's just presentation - it's to make the maths work.

Algebra and Addition
The Amazing Coincidence

But there is more mathematical mystery to follow. When you get them to write this number backwards and add, what you're really doing is adding

A-1-C	9	10+C-A
+ 10+C-A	9	A-1-C
9	18	9

We can simplify this. That 18 in the middle means 180 (remember it's in the tens column) so we can take the 100 from 180 and shift it to the hundreds column, leaving us 8 in the middle. So the final sum is:

10	8	9

Which of course is 10 hundreds, 8 tens and 9 units or 1089 - the number you had predicted all along!

Algebra – The Applications

Algebra is a wonderful tool for testing and predicting our understanding of the world. The *Xs*, *Ys* and *Zs* in the algebra mean something real. Algebra lets us take a word description of the world and change it into a mathematical description which is really useful. Clothing designers use algebra to work out how best to cut cloth, engineers use algebra to design cars, boats and aeroplanes and the next generation of medicines customised to our genetic individuality will use algebra. Plumbers and carpet fitters use algebra to work out how to cut pipes or carpets to fit a space and the financial sector uses algebra to predict the money market.

Prime Numbers
Twice the Impossible Location

You cut the deck of cards in half, passing one half to one spectator and the second to another. You then ask them to take a random card out of their part of the deck, memorise it, and then pass it to the other person. Each spectator places their card on top of their pile, and then shuffles it thoroughly. The two are combined, and you quickly scan through the deck and select the two chosen cards.

You need some set-up for his trick. At the start, the deck of cards is secretly separated into two groups - one set of cards with values that are prime numbers and one set of non-prime values. By doing this, you will easily be able to tell exactly what cards were swapped but your spectators won't notice.

So start by dividing up the deck with all the prime numbers in one stack (2, 3, 5, 7, J, K) and the non-prime in the other (A, 4, 6, 8, 9, 10, Q). Keep the suits of the cards in a nice random order as it's only the value that's important here. You can put a Joker between these two sections to make it easy to separate the two sets.

When you begin your performance, spread the cards showing they are 'random', then 'notice' the Joker, take it out and split the deck at that point. Give one half to each spectator. They do their selecting and swapping, but when you get the cards back you simply look for the card that's out of place in the respective sections. There will be a prime valued card in the non-primes pile and via versa. Use your performance (and mathematical) skills to reveal their selected cards.

You could present this as a demonstration of your super-powered sense of smell. When the spectators select cards, you tell them they have left their scent on them. Then while you're scanning through the pack to find say the prime card in the non-prime pile, make lots of loud sniffing sounds. This will give you plenty of misdirection to check the cards and may get a laugh too!

You will remember that a prime number is a number that is only divisible by itself or one – we say it has no other factors. That's what makes them different, and that's the mathematical trick you use here.

Is 1 a prime number?
You will notice that the Ace, which represents the number 1, is in the non-prime group. To be prime, a number needs to have both itself and one as factors, and the number 1 can't have both! But don't worry, it took mathematicians years of arguing to decide that one is prime, so if you find it a bit strange, you're in good company.

If a number is not prime, then we call it a *composite* number. Clearly 1 is not composite, as it doesn't have any *factors*. So if you find it easier to remember, then have one group of (4, 6, 8, 9, 10, Q) for the *composite* numbers and another group (A, 2, 3, 5, 7, J, K) for the *non-composite*. As long as you are consistent, the magic works the same!

4	9	2
3	5	7
8	1	6

The Classical Lo Shu Magic Square

17	89	71
113	59	5
47	29	101

A Magic Square using prime numbers

Prime Numbers – The Applications

Mathematicians see prime numbers as the 'atoms' of mathematics. Just as all matter in the universe can be built up using physical atoms, all numbers can be built up by multiplying the appropriate primes. The properties of prime numbers are central to the encryption methods used for Internet shopping, and prime numbers have inspired musicians and artists in creating their music, art, literature and films. Prime numbers can appear in nature too. Some biologists believe that insects called *cicadas* use prime numbers as a survival strategy. These insects live their lives underground but emerge every 13 or 17 years to breed. The fact that they only emerge in prime numbered intervals makes it hard for predators to evolve to feed on them as it is difficult to predict when they'll emerge. Doing the maths, scientists show that this prime number trick improves their survival rate by a few important percent.

Geometry
The Find a Card by Psychic Aura Trick

You explain that every person is surrounded by an aura, a psychic energy field that is generated by the 'life force'. You then go on to explain that when you touch an object, part of your personal aura rubs off, like leaving a fingerprint.

To prove you can detect this residual aura you deal cards one by one from the top of the deck, face down in a line on the table, until your spectator tells you to stop. Have your spectator pull one of the cards - their free choice - out from the line, keeping it face-down on the table.

While you collect the remaining cards from the table, have them take a sneaky peek at their card by just quickly lifting the end so only they know the value. Remark that they are also marking it with their aura. They slide their chosen card back into the pack and you allow them to shuffle so their card is lost. You then take the pack and using your aura detecting skills dramatically reveal the card they chose.

The aura story is a fake. This trick is based on the geometry of so called *pointer cards*.

These are cards that do not look the same when rotated through 180 degrees. Look through a pack. First off, all the picture cards (Kings, Queens and Jacks) look exactly the same when you rotate the pack. And most Diamond cards look the same upwards and downwards as the geometry of the shape is the same under a 180 degree turn (the 7♦ and the 9♦ are, however, *pointer cards*). Now look at the Spades, Clubs and Hearts. These shapes are different when they are rotated, for example, Hearts only have a point at one end. When you turn the cards round the Hearts point in another direction.

This property of geometry, *shape symmetry*, is the key to the trick. In a standard deck there are 22 pointer cards. For example the Ace of Hearts, Clubs and Spades (but not Diamonds!), the 6♥, the 5♣, the 3♠, and even the 7♦ (because of the way the Diamond pips are laid out) all look different in one direction to the other.

Here it's important that you manage what the spectator does carefully. The handling of the card selection and returning to the pack need care as the direction in which your pack is facing is crucial.

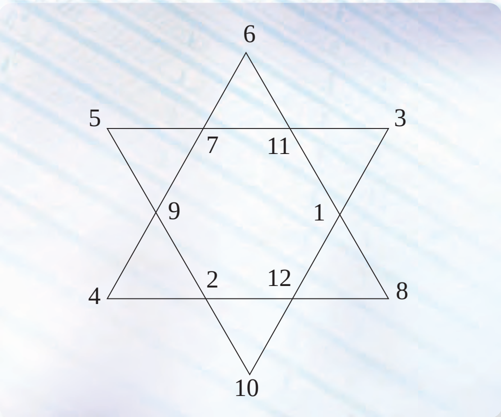
Before the trick, you stack the 22 *pointer cards* on top of the deck, all pointing one way. Remember that direction! Your spectator must choose their card from this first 22 otherwise there will be trouble. Most spectators will feel under pressure and so won't go past ten or so. If the spectator looks like they are going well past that, remind them they can stop at any time - they will take the hint!

Have them take a sneaky peek at their card by just quickly lifting the end so only they know the value.

As you deal, you are carefully keeping the cards aligned in the same direction as your original stack. So it is very important that the spectator does not turn their card around when they are looking at it, and that is why you get them to peek at it on the table rather than picking it up.

Meanwhile, you carefully collect all the other cards from the table, and while your spectator is busy, turn the cards round in your hand. When your spectator then slides their chosen card into your pack, their card will be the only one 'pointing' the other way round.

They can overhand shuffle to their heart's content, it won't turn any cards round. But make sure they don't do any funny shuffles where they split the pack and so may turn sections round. To control the shuffle, simply mime the shuffle you want them to do. It's then easy for you to spot the reversed card in the pack as you fan through since your brain is good at recognising shapes. You can now reveal the chosen card.



The clever geometry of a Magic Hexagram for the number 26

The design of spectacular stage illusions needs a good understanding of trigonometry and geometry to calculate audience sight lines and work out the best ways to engineer 'hidden' spaces.

Geometry – The Applications

Geometry is used in many different jobs. Architects and designers use the laws of geometry to create stunning new buildings and products. Without it they wouldn't know what shapes to use or how they would fit together. Stained glass window designers and fashion designers also use geometry in their work as certain geometries look pleasing to the eye.

Geometry is behind most of those screen busting special effects in movies, TV shows and video games, so no surprise that massive numbers of the people employed in the games industry have maths or computer science degrees.

Geometry and symmetry are also important in the pharmaceutical industry. Being able to work out the shapes of new chemicals and how they fit together with molecules in our bodies is central to discovering new life improving drugs.

Statistics Quick Think of a Number, The Street Magic Stunt

Street magicians like David Blaine often use this psychological trick. Ask your friend to quickly think of a two-digit number between 1 and 100, both digits odd and both digits different from each other. Concentrate, the answer is 37!

First up, this trick doesn't always work as it's not strictly mathematical! Of course in the TV shows they only show the time it did work. It's based on *probability* and a rather sneaky way of reducing your spectators' choice. If it goes wrong for you, hey, it's mind reading, it's supposed to be hard. You were just not tuned in properly...

$\sqrt{x}$

You start by saying they can choose any two-digit number between 1 and 100. They will remember you giving them that 1-100 choice, it's called the *primacy effect in memory*, which means that you tend to remember the things at the start better. Two digits means 1-9 are eliminated instantly, but then you go on to say both digits must be odd. Now more than half the remaining possible numbers are gone too: all the even ones and all the ones that start with an even digit.

Then you say both digits must be different. This narrows it down even more. There are in fact very few numbers left that the spectator can choose from though they don't tend to notice this.

This is where the statistics comes in. When asked to give the number quickly, the vast majority of people will say 37. It may be that it is somewhere in the middle - 13 would be too small, 97 would be too big. It may be that the numbers 3 and 7, which themselves are the most common answers if you ask people to name a number between 1 and 10, just seem to come together. Whatever the reason there is an increased chance that you get the 37 you want.

Mathematicians Persi Diaconis and David Bayer have shown that you need to give a pack of cards seven dovetail shuffles before the cards are really in a random order.

Martin David Kruskal, creator of the statistically based Kruskal count card trick, was an American mathematician and physicist. He also worked on general relativity and the theory of solitons.

Statistics –
The Applications

Statistics is a branch of mathematics that lets us describe random things, like coin tosses, roulette wheels or radioactive particle decay. Some people think it is just about averages, but without this sort of mathematics we would be unable to carry out a range of really important tasks. Statistics is used in many jobs. Here are some examples:

- manufacturing: to work out the probability a light bulb will fail or a pipe might leak
- drug companies and doctors: to help discover if a new drug works
- TV companies: to work out viewing figures
- banks: to model the markets
- bookmaker: this job is based almost entirely on statistics, for example, in calculating the odds in a horse race.

Advanced Magic and Mathematics

The following tricks make use of some advanced mathematics you may not have seen before. It's called *clock arithmetic* (the posh name is *modular arithmetic*). You will, however, be familiar with the idea, unless you only wear digital watches. In clock arithmetic we count as if we are going round a clock face.

A normal clock has the numbers 1 to 12 on it. If we start at 1 o'clock and count forward an hour we get to 2 o'clock. Now suppose we start at 1 and count forward 12. We don't end up at 13, we end up back at 1 as the count has looped round.

That's the idea of modular arithmetic: it's about repeating cycles of numbers. A normal clock has 12 numbers and so arithmetic on such a clock is called *Modulus 12*. But we can think of other stranger clocks with any numbers we fancy. For example, doing clock or modular arithmetic at *Modulus 3* would be like having a clock with only 3 positions. Starting at 1 and counting up we go 1,2,3,1,2,3,1,2,3 and so on as we go round and round. We normally include a 0 position in the clock, so with Modulus 3 rather than 1,2,3 we have 0, 1 and 2 and the count goes 0,1,2,0,1,2,0,1,2.

Clock Arithmetic The Fairest Test (ever) of Psychic Skills

This is a strong effect. You select a spectator who believes they have good intuition, and proceed to test it using a 'well known psychic test procedure' with some cards. You have a cut-down pack containing just Ace to 5 in a red suit, and Ace to 5 in a black suit. The spectator mixes the cards then you deal two piles of five.

You will eliminate pairs of cards by letting your spectator make free choices. And then they have to use their intuition to make sure the last two cards left match, for example a red three and a black three. After following the fair rules of the psychological test, you show their intuition was correct - the last two cards match!

But as their jaw drops, you show them that they are even better: every pair of cards they eliminated, under completely free conditions, and following the test rules, match in value! This is a show stopper and a good finale to your act.

Explain that this is a well-defined test with strict rules to follow which will score people's psychic ability. Start with Ace to 5 in a red suit and Ace to 5 in a black suit. Pile these cards in sequence on top of each other, that is, A to 5 red then A to 5 black. Spread the cards face down and have the spectator point to the back of one of them. Cut the deck at that point.

Continue with this until the spectator is happy the cards are well mixed. Then the secret move, oh so simple... Deal the top five cards onto the table, reversing their order, and place the remaining cards beside them. You explain they have five cards in each pile and that the test conditions will give them four chances to use their intuition at this stage. They have four swaps, no more no less. (A swap involves taking the top card on a pile and placing it on the bottom.) Explain they can perform all four swaps on one pile, or two on each or three on one and one on the other.

Once the four swaps are made, remove the top card on each pile and place them aside. Now there are four cards in each pile. The test rules say you now offer the spectator three swaps. Do the swaps they indicate, and then remove the top two cards from the two piles. Now there are three cards left in each pile and the rules say two swaps. Carry these out and then remove the top card from both piles, leaving two cards in both piles.

Every pair of cards they eliminated, under completely free conditions, and following the test rules, match in value!

Now it's the final chance, one swap, and one card can make all the difference. Play this up. They choose their swap, the top two cards from each pile are discarded and the final two single cards on the table are revealed. They match! The spectator's intuition worked even though they had a free choice on all the card swaps. But there is more - as you dramatically reveal all the pairs of cards that were removed match in value. Wow, they have a massive ESP score!

Clock Arithmetic The Fairest Test (ever) of Psychic Skills

Believe it or not, follow the instructions above and the trick works automatically. Let's see why. At the start we have the first pile of cards in order A2345A2345. Think of the stacked pack as being circular, the cards A to 5 being positions on a clock 1 to 5. Cutting these cards with a single cut doesn't disturb this cyclic order. So for example, a cut between 2 and 3 may move the card order to 3451234512, but the 12345 and 12345 of the original is still there – just as when we count round we will still get 12345 followed by 12345. The sequence just starts at different 'times' on the clock face.

For simplicity, let's assume the cards are in the original order. Our secret move, counting five cards reversing their order gives us 12345 in one pile and 54321 in the other. This is called a *palindromic stack* (same forward as backward) as the values in one pile are exactly reversed in the other.

To make our explanation even simpler, consider only three card values so our piles are 123 and 321. These are palindromic. Now make two swaps (remember the number of swaps is always one less than the number of cards in the piles). What are the possibilities? Two swaps on the first pile, top card to bottom and again, takes us to 312, and the top card matches with the 3 on the other pile as we require. If the two swaps are made one on each pile, the first pile goes to 231 and the second pile to 213 and the top cards again match. Remove the top cards, leaving 31 and 13, so whatever pile the final swap is made on, the removed cards will match and the remaining cards too.

This mathematics works for any number of cards N in order in palindromic stacks, where $N-1$ swaps are made. As one card is moved down, so the other rises up in the other stack and they match. This is another property of *clock arithmetic*, or *modular arithmetic* - all the cards are in a circular order, but in one pile the clock runs clockwise whilst in the other pile it runs counter-clockwise.

Clock Arithmetic
Professional Deck Stacking

Magicians often work their wonders with cards by putting them in a specific order that looks to their audience to be completely random. This is called *stacking the deck*. The following stacked decks take a bit of time to set up and you will need to learn the secret order which is the mathematical key. But once you have learned the mathematical trick the rest is easy and the results are marvellous. This is a professional magic technique with a long and distinguished history, so guard the secret well.

A mathematical stack lets you work out the position of any other card in the deck just by secretly looking at the bottom card of the deck. The sorts of magical effects you can use this for are left to your imagination but a few are given here as examples. Stacked decks go way back in history. Here we will cover two of them, a simple arithmetic approach called the *Si Stebbins stack* and another, slightly more challenging classic, called the *Eight Kings stack*. Both these stacks make use of *CHaSeD* order which we met earlier on. But they use different secrets to hide information on the card values. So let's start by looking in more detail at CHaSeD order.

***CHaSeD* order**

CHaSeD is our mnemonic for the order of card suits: Clubs, Hearts, Spades then Diamonds. It's cyclic, so then it's back to Clubs. This order is easy enough to remember and both of the following stacks make use of this pattern.

The Si Stebbins Stack

This useful card stack system was originally published in the United States around 1898 by William Coffrin in his booklet entitled *"Si Stebbins' Card tricks and the Way He Performs Them"*. The name of the stack comes from Coffrin's stage name which was, not surprisingly, Si Stebbins. You will set the suits in *CHaSeD* order with each card having a value three more than the card before it in the stack. It is simple arithmetic.

Setting the deck in Si Stebbins order

There is an easy way to set up this stack. Start by separating the suits in the deck. Now order each suit numerically in a face-up pile, Ace at the bottom up to the King visible at the top. Set these four piles of cards on the table in front of you in order, left to right: Clubs, Hearts, Spades, Diamonds (our now familiar *CHaSeD* order).

Leave the Clubs pile alone. For the Hearts pile, leave the Ace, 2 and 3 in the same order, but move them from the bottom to the top so you can see the 3. For the Spades pile, take the Ace to 6 in order and move them to the top of the pile. Similarly, with the Diamonds pile shift the Ace to 9 in order to the top. Each pile is therefore reordered with a relative shift of none, three, six or nine cards as we go left to right.

You should now have the four *CHSD* piles in front of you, with the King, 3, 6 and 9 cards visible, left to right. To create the stack simply start with the Club pile, showing the King of Clubs, then take one card from each of the suits piles in turn, working left to right, and stack them face-up on top of each other. When you reach the Diamonds pile go back to the Clubs pile and do it all again, repeating till all the cards have been used. Once you have finished, turn the deck over to the normal face-down position.

Done correctly, the first few cards from the top down will be the K♣, 3♥, 6♠, 9♦, Q♣ and so on. In other words, the cards are in a suit order that repeats *CHSDCHSD* and each following card has a value of three more than the previous card in the sequence. You will notice the 5♣ is followed by the 8♥ (as 5+3=8); the J♠ (value 11) is followed by the A♦ (although 11+3 =14, we only have 13 card values so following our clock arithmetic, we're back to the beginning).

A simple trick with Si Stebbins

Here's an easy and simple effect with this stack. You can cut the cards as many times as you like, but we know this won't disturb the cyclic stack order. However, don't shuffle them!

Ask someone to cut the deck at whatever position they want to form two piles. You pick up the pile that was originally the top half of the deck and ask the spectator (or even another spectator) to take the top card from the other pile which is the card which was cut to.

As you recombine the two piles, you casually glimpse the bottom card of the top pile. From this card value you will know what card was chosen. It will have a value of three more than the bottom card you glimpsed and its suit will be the next suit in the *CHaSeD* order. For example, you glimpse the 7♠ on the bottom and predict the 10♦ as the freely chosen card.

The Eight Kings stack

While it's a great stack, the regular plus 3 rule of the card values in Si Stebbins can very occasionally be spotted by the audience especially if you ever spread the cards to show they are 'random'. So some magicians prefer to use another method. A deck stacked in Eight Kings order looks like this.

8♣, K♥, 3♠, 10♦, 2♣, 7♥, 9♠, 5♦, Q♣, 4♥, A♠, 6♦, J♣, 8♥, K♠, 3♦, 10♣, 2♥, 7♠, 9♦, 5♣, Q♥, 4♠, A♦, 6♣, J♥, 8♠, K♦, 3♣, 10♥, 2♠, 7♦, 9♣, 5♥, Q♠, 4♦, A♣, 6♥, J♠, 8♦, K♣, 3♥, 10♠, 2♦, 7♣, 9♥, 5♠, Q♦, 4♣, A♥, 6♠, J♦.

To the audience, and you too probably, this really looks like a shuffled, fair and random deck, but there are still patterns in here that let the magic happen. Mathematicians like a pattern - that's what maths is really all about, discovering the patterns in the world around us. But in this case we have created the patterns to do a job. Can you spot any familiar patterns in this deck?

Setting up the Eight Kings stack
First let's create the stack, so we can experiment with it. Get a complete pack of cards without jokers. First, put the 13 cards of each suit face up in order in four piles, highest (King) on top this time. You are going to set the order of the suits to repeat the familiar Clubs, Hearts, Spades, Diamonds cycle again (did you spot that in the deck listed above?). You are going to set the values of the cards following the pattern 8, K, 3, 10, 2, 7, 9, 5, Q, 4, A, 6, then the J.

What about the values, what's the pattern there? There isn't an obvious arithmetic one, it's just a random order of cards. The secret lies in the phrase ***'Eight kings threatened to save ninety five queens for one sick Jack'***. Make a vivid mental picture of the scene in your head, those Kings, saving all those Queens and that one sick Jack.

What's that all about? Well it's another mnemonic. The phrase is a way of remembering the pattern. It breaks down as Eight Kings (8, K) ThreaTened (3, 10) To (2) Save (7) Ninety-Five (9-5) Queens (Q) For (4) one (A) Sick (6) Jack (J). There you have it, a pattern of card values your audience won't spot, and easy for you to remember.

Put the suits and values together, card by card 8♣, K♥, 3♠, 10♦, 2♣, etc, and your deck is stacked. Mathematically this set up is still a cyclic stack, even though there isn't a simple arithmetic key to reveal the card values. Like the hands on a clock, the cards run through the same suits and values again and again, but we have two 'clocks' working here.

The 'clock' face for the suits contains only four positions: Clubs, Hearts, Spades and Diamonds. As there are four values, we call this *modulo 4*. The suits 'clock' has 13 positions, in order 8, K, 3, 10, 2, 7, 9, 5, Q, 4, A, 6, J, so it counts in modulo 13, as after a J it's back to an 8. So, now, how do you now tell the position of any card from just glimpsing the bottom card?

One possible trick is to have a spectator cut the deck (again, simple cuts from top to bottom won't disturb either of the cycles in the stack, the cards still go round and round). You secretly look at the bottom card and call or predict the next up in the Eight Kings cycle in the next suit from our stacked *CHaSeD* sequence. You now know the top card, and you can simply step forward in the stacks to know the card under it, and so on.

Do you know what card your spectator will choose? No, and neither does anyone! But you know that the deck is stacked with repeating clock-like patterns. A Spade will always follow a Heart, a 10 will always follow a 3 and so on.

The Eight Kings stack

Perhaps build the presentation a bit. Start with trying to predict the top card suit only and maybe even get the first one wrong on purpose. This is called a *miscall* and can help create tension - after all, mind-reading can't be that easy. Throw the top card away in 'disgust' then concentrate on the second card, get the suit right, and show it, then perhaps the suit of the third, get that right but be one or two off on the card value. Then finally, once you're tuned in to the 'psychic vibrations', you can correctly identify the next few cards. Don't overdo it though - the audience may spot the repeating pattern in the suits if you do, and bang goes that all important astonishment.

Where in the pack?

With practice, you can calculate the value of any card at any position in the deck because all you need to know is the bottom card. So, ask a spectator to give you a number between 1 and 52. Let's say it's 17. Now you can tell them the value of the 17th card in the deck to impress them.

$\sqrt{x}$

If the bottom card is a Spade, the top card will be one forward in the *CHaSeD* stack which is a Diamond. The second card down will be a Club and so on. Every fourth card from the top Diamond will also be a Diamond, as the suits cycle round and back to their start in steps of fours, so the card 16 from the top will be a Diamond, and the next card, the 17th, will be a Club.

It is easy to work this out using a formula. First move forward in *CHaSeD* order by 1 to find the suit of the top card – here it's Diamonds. Next, we divide 17 by 4 (the number of suits) to get 4 remainder 1. All that's important here is that remainder 1, and it tells you to add 1 to the suit of the top card, our Diamond, to give a Club.

This formula will work for the suit of a card at any position. Glimpsed the K♣ and want to tell the suit of the 26th card? Step 1, take one step forward in *CHaSeD* to work out the top card is a Heart. Step 2, divide 26 by 4 to get 6 remainder 2. Add 2 to give the card suit Diamonds.

This dividing by 4 and keeping only the remainder is called *modular arithmetic*. But what if you're given a small number, say 3? It's easy: 3 divided by 4 is 0 with remainder 3.

Once you're tuned in to the 'psychic vibrations', you can correctly identify the next few cards.

Similarly you can predict the value of the card at any position. This time the division of the given number is by 13. We follow the same rules, so suppose we want to know the value at position 28. Well, 28 divided by 13 is 2, remainder 2. Now remember your *Eight kings threatened to save ninety five queens for one sick Jack* or 8, K, 3, 10, 2, 7, 9, 5, Q, 4, A, 6, J. If the bottom card is a 7, then the top card will be a 9, and to get the chosen card we just we move 2 forward from the 9 in the stack to discover the value of the card is Q.

Combine this with the earlier suits formula and you can calculate the position of every card in the deck. It may take some practice to learn this, but when you do, you have some really strong magic techniques.

The Eight Kings stack

Other quick tricks using Eight Kings Stack
Ask the spectator to cut the pack a few times. Pretend to memorise these cards instantly by spreading them, but just remember the bottom card. You can then call out card names at various positions in the deck, or tell spectators where their 'favourite' cards are in the deck.

Or even easier, ask someone to pick any card, look at it and remember it, and then to place it back into the deck anywhere they wish (make sure it's not put back in the same place). You can give the cards some straight cuts then simply spread the cards and search through looking at the patterns to find the card that is out of place.

You can of course do similar tricks with the Si Stebbins stack, and you may find that stack easier to start with as the arithmetic can be easier. With Si Stebbins, the suits formula is exactly the same as before, but for the value, remember to add a 3 to work out the value on top of the pack from that all important bottom card you have seen. Then you just need to keep adding multiples of 3 to get card values. But watch out, that simple plus 3 step in card values may be spotted. Try both stacks and see which works best for you; you will have to practise a bit with both, of course, to get the maths embedded in your head.

Stacking it all together
So we have in our stacks some clever mathematical modular arithmetic forming a traditional magician's secret stack. The suits of the cards are related by modulo 4, (1,2,3,4,1,2,3,4...) and our card values are related by the sneaky Si Stebbins or Eight Kings cyclic stack, which also repeats modulo 13. Using this mathematical 'model', you have a way of mathematically describing the position of every card in the deck even though, to the untrained eye, it looks random. All you need to know is the bottom card and, with practice and a compelling presentation, this can lead to some mathematical miracles.

The simple beauty of Palindromic numbers

1 x 1 = 1

11 x 11 = 121

111 x 111 = 12321

1111 x 1111 = 1234321

11111 x 11111 = 123454321

111111 x 111111 = 12345654321

1111111 x 1111111 = 1234567654321

11111111 x 11111111 = 123456787654321

111111111 x 111111111 = 12345678987654321

Modular arithmetic
The Applications

Modular arithmetic is one of the mathematical steps in perhaps the greatest wonder of the modern technical world - the internet. We are all aware of online shopping and we would all hope that our personal communications by mobile phone or chat room would remain private from eavesdroppers. Well, at the centre of the encryption technology that allows the secure transmission of bank or credit card details, or that ensures the digits making up your phone calls don't fall into the wrong hands, there is some clever modular arithmetic.

Using a very large Modulus means that it is almost impossible for an eavesdropper to go back and discover what the original numbers were as they don't know how many positions there were on the 'clock face'. So exactly the same mathematics that secretly stacked the card deck also helps preserves our security on the internet. It is also used to help keep chemicals safe, as each type of chemical compound in the world is given a unique code number, called a *CAS identifier*. This ensures that people know 'what's in the test tube'. This code number uses modular arithmetic as a check to help ensure that any errors in labelling are corrected. Modular arithmetic also turns up in music and is used to help tune musical instruments.

Final Words

This exploration of the *Magic of Mathematics* is now at an end, but there is so much more out there to discover. Hopefully, you will have learned some amazing magic. But also, we hope you have learned some amazing mathematics, how it can be applied and how important it is to almost every job in the world.

The rest is up to you. Practise the tricks and your patter, build your own presentations, improve the ideas, be entertaining, create astonishment and, most importantly, enjoy the wonders of mathematics. Mathematics is the real magic that makes our world work, whether it's helping you do a trick, designing new medicines, buildings or cars or even if it is hidden in your games console, PC or mobile phone.

You may just want to learn some more magic. That's great. But hopefully you will also want to learn some new mathematics too. There are some great books on mathematical magic which we can recommend. These classic magic books mostly just describe the effects, so it will be up to you to discover the hidden mathematics. It will be worth it because once you know the mathematical secrets, you can apply your own creativity and come up with new presentations. Who knows, maybe in the future you might be fooling magicians too!

1. *Magic of Computer Science* Two free books of magic tricks, based on mathematics and computer science can be downloaded at www.cs4fn.org. Some of the tricks in this current book are derived from this series, but there are a whole lot of extra tricks, using other types of maths and psychology to amaze your audience. Well worth a read.

2. Martin Gardner *'Mathematics, Magic and Mystery'*, Dover Books. A whole range of mathematical card tricks, coin tricks and number tricks, this book is entertaining and has plenty of explanation. A must have classic from one of the best.

3. Karl Fulves, *'Self-working Card Tricks',* Dover Publications Inc. Karl Fulves is a famous magician, and this book focuses on self-working mostly maths-based tricks, but also introduces some simple sleight of hand tricks. Great for the beginner. Read the book, entertain your friends and work out the maths secrets for yourself.

4. Karl Fulves *'Self-Working Number Magic: 101 Foolproof Tricks,* Dover Publications Inc. This book is full of speed calculations and all sorts of number-based magic tricks. Well worth a read to find new tricks and explore the mathematics behind some amazing number illusions.

The author Peter McOwan appears courtesy of the CS4FN (Computer Science for Fun) project www.cs4fn.org.

This book has mathematical secrets...

This book has mathematical magic secrets built in to help you with your trickery.

The back cover of the book has the prediction number for the *Amazing Coincidence 1089* effect described in the book hidden as a serial number. The inside cover contains a ‘This is your card’ picture. Use the card shown as your force card and you can use this picture to reveal your prediction. Details of card forces can be found in the book. The book front cover also has a design containing the suits in *CHaSeD* order and also the *Eight Kings* stack to help you remember these.

For further information and instructional videos please visit:
www.mathematicalmagic.com

If you would like to get in touch then please email us at:
mail@mathematicalmagic.com

Remember the Magicians Code: practise, practise, practise. And never reveal the workings of magic tricks to your audience!

Bonus effects The Maths Hustle (or How to Win with Maths)

Hidden away here at the end are a few extras. These aren't magic effects, they are maths hustles, tricks and ways you can use mathematical principles and a little bit of cunning to always win games. Use them responsibly and add them into you magic act for a bit of variety!

Bonus Hustle effect 1
The Glass Challenge

You challenge your friends to a friendly wager

It’s a challenge where it looks like you can’t be right, but in the end you triumph. On the table is a normal drinking glass. The wager is this. Which is longer: the distance from the table to the top of the glass or the distance round the rim of the glass?

They make their guess. You then pull out a full pack of cards, and put the glass on top of the cards. Again the question is the same: which is longer, the distance from the table to the top of the glass that is now on top of the cards, or the distance round the rim of the glass? They will probably start to change their guess.

You pull out another deck of cards. Put that on top of the first deck and then put the glass on top of them both. Again you challenge them: which is longer, the distance from the table to the rim of the now rather elevated glass or the distance round the rim of glass?

It’s too obvious!! (Or is it??) They all change their guesses. You alone stick to your guns: the distance round the rim is longer.

In the final pay off, you produce a piece of string you ‘prepared earlier’. It fits snugly round the glass rim. Then, to your spectators’ amazement, when you use it to measure the distance from the table top to the rim of the glass placed on the two staked card decks, it’s longer! The round the rim distance is longer as you alone still predicted!

There is no trick. It’s just geometry! For this stunt we count on two things. Number one is that people don’t remember a lot of the maths they did at school. Number two is simply an optical illusion. You need to test this out first with some common glasses to see how far you can go with each. Plastic cups from a drinks machine may work best. Choose something your spectators will be familiar with.

Bonus Hustle effect 1
The Glass Challenge

The distance round the rim (the circumference) C, is related to the glass's diameter d by the formula $C = \pi \times d$ (remember, π is the number 3.14159). So that means that the distance round the rim is a bit over three times the diameter. Most glasses expand towards the top, so the circumference is actually quite large. But not a lot of people know that. It's basic maths, but often forgotten.

You have a head start here then. People don't realise that you have a factor of three times the glass diameter to play with. That's where the optical illusion comes in. If you show people a letter T, where the top horizontal line and the vertical line are in reality the same length, people perceive that the upright vertical line is longer. It's another of those brain errors!

If you think of your glass now seen from the side, the diameter at the top will be seen as being shorter than the distance from the table to the rim. That will happen even if they are the same distance. Combine this optical illusion with the maths mistake about $C = \pi \times d$, and you will find that with most glasses you can stack them on a range of easy to hand objects, and the distance round the rim, the circumference, will still be longer.

Do check your glass before you perform though, and make sure you know what you can stack it on to still win the challenge. Having a bit of string to prove the point makes the finale easier and punchier. Most important of all, play the spectators! Stick a couple of packs of cards down on the stack. Then even if there is still someone not 100% sure, 'grudgingly' add another pack, making it look like you have lost the challenge. Then it's out with your string for proof that you are the winner. Who said geometry wasn't useful!?

Bonus Hustle Effect 2
The Napkin Game

Take a paper napkin, open it out, fold the two opposite corners together and tear off the excess to make a perfect square. Fold the other two opposite corners together. This way, when you unfold it they will have a square with the centre position marked where the diagonal creases cross. Then you offer to play the game.

Get a pile of coins, all the same value - two-pence pieces are good for this. The challenge is that you and your opponent will take it in turns to place a coin on the napkin. The loser is the person who can't place their final coin without it going over the edge of the napkin.

Now the first trick. Ask your opponent if they want to go first or second. If they ask you to go first, place your first coin exactly on the middle of the napkin square. If they say they want to go first, then 'remind' them of another rule of the game: to begin there is a coin placed exactly on the centre and then it's their go. Either way there will be a coin in the middle, right over the central creases, and they will have to make the next move.

You have won already! The rest of the game is just you and your knowledge of symmetry. Wherever your opponent places a coin, you place your coin symmetrically around the centre coin, on the opposite side. For example if they put a coin 1cm away from the centre coin at the 12 o'clock position, you place your coin 1cm away from the centre coin at the 6 o'clock position.

Since they have to always place their coin before yours (because you ensured that centre coin was in position and that they would play right after it was placed) and you mirror their moves, they must run out of coin space on the napkin before you do. It's simple symmetry. Fancy a game?

Glossary of useful terms

Algebra The word algebra is believed to come from the Arabic word *al-jabr*, which many scholars believe refers to the act of restoration or balancing. In algebra, the equations contain an equals sign, and the rule is what you do to one side of an equation, you do to the other side too, keeping that balance.

Astonishment The audience's state of mind you're aiming for. Even if the secret behind the trick is simple, the correct presentation can leave them feeling amazed. They have seen the impossible made possible. If you get this to happen leave them to ponder it for a bit before you move on.

Binary A way of counting that uses just 1 and 0 rather than the decimal system's 0,1,2,3,4,5,6,7,8,9. The binary system uses powers of two: 1, 2, 4, 8, 16, 32 etc, to represent numbers. For example binary number 101 represents one 4 + zero 2s + 1 unit = 5 in decimal. Binary, is the mathematical language used for electronic computers as it can be represented as *on* and *off* switches.

CHaSeD A memory aid (a *mnemonic*) to remember the cyclic order of card suits, the capital letters here representing the order of the suits, Clubs, Hearts, Spades and Diamonds.

Clock arithmetic A way of adding and subtracting where the numbers used are imagined to be arranged at positions on a clock face and so repeat depending on the number of positions. For example, counting round 12 positions on a standard clock face takes you back to the number where you started. If the clock is imagined to have only four positions any multiple of 4, say 8 or 16 will take you back to the same starting position. The number of positions imagined on the clock, i.e. the cycle through which you pass to come back to the start, is called the *Modulus* of the numbers.

Cyclic order A pattern of cards that repeats itself, for example 234 followed by 234 followed by 234 and so on. See *CHaSeD* and *Eight Kings*.

Eight Kings A traditional magician's mnemonic for remembering a card stack. It stands for Eight Kings (8-K) ThreaTened (3-10-ed) To Save (7) Ninety-Five (9-5) Queens (Q) For (4) one (A) Sick (6) Jack (J).

False Cut To appear to mix up the order of the cards by cutting the deck into smaller portions and reassembling but really you retain the cards in order.

Force To ensure your spectator chooses the card you want them to while they think their choice is completely free.

Glimpse To secretly look at a card, for example the card on the bottom of a face down pack, so that your spectator doesn't notice.

Mnemonic A memory aid that helps you remember something. Often a word or phrase where the letters help to represent the information to be remembered in the correct order, for example see *Eight Kings* or *CHaSeD*.

Modular arithmetic see *clock arithmetic*

Overhand shuffle Standard shuffle for cards where the spectator or magician takes cards from the back section of the pack and moves them to the front time and time again.

Palindromic Refers normally to a word or sentence that reads the same forward as backwards, for example 'Sit on a potato pan, Otis'. It can also apply to numbers for example 123321.

Patter A term for the story you tell as you do the trick. Remember, a trick should be easy to follow and make sense. Is it an attempt at mind reading? If so, tell them that is what it's about and have a story that possibly makes it seem like an experiment that could go wrong - audiences like that kind of jeopardy.

Practice They say it makes perfect and magic is no exception. Practise until you are comfortable that you can do the moves correctly but also that you can talk an interesting storyline for the trick to keep the audience entertained. See *patter*

Prime A prime number is a number divisable only by itself or 1. Prime numbers are often called the 'atoms' of maths as they can be used to build other numbers. They also have useful applications, for example, in data encryption.

Si Stebbins order A classical full deck card stack where the suits of the cards cycle in CHaSeD order, and the values of the cards count up in steps of three.

Stack Placing a deck of cards in a certain secret order to allow you to perform a trick, for example see *CHaSeD*.

Showmanship One of the key ingredients of magic. Learn the secrets, practise the moves, but present them in an entertaining way. Work with your own personality to make it right for you, and each time you perform, learn how to make it better next time. A trick should have a well-defined beginning, middle and end, like any good story. Keep the presentation as simple as possible, that way the next day your audience will be able to tell others what happened.

The final page for you to add your own mathematical magical creations

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